RELAXATIONAL FILTRATION

A. Yu. Zubarev and B. Khuzhaerov

UDC 532.546

The relaxational phenomena arising in the filtration of real liquids due to silt deposition and suffosion by pores by particles suspended in filtrate are considered.

It is known that nonsteady filtration of polymer solutions, heavy petroleum oils, and suspensions is accompanied by a series of relaxational effects [1-4]. The investigation of such processes is important in connection with many applied problems and, in particular, with problems of petroleum extraction, amelioration, acoustic probing of liquid-saturated beds, etc.

Relaxational phenomena in filtrational processes are usually taken into account by a purely phenomenological generalized Darcy's law, by assigning as relaxational form to it and introducing of the characteristic times, which are regarded as empirical constants [4-7]. However, for correct interpretation of relaxational processes, control of them, and a priori prediction, it is necessary to relate the structure of the correpsonding equations and the values of the parameters appearing there to the physical properties of the filtering liquid and the porous material.

There may be very different reasons for the relaxational behavior of the filtrational fluxes. It is natural, first of all, to relate their appearance to elastoviscous properties of the filtrate. However, experiments show that the characteristic times of the filtrational fluxes, as a rule, are much greater than the corresponding values in viscosimetric flows, and depend strongly on the structure and physical properties of the porous medium; the values of these times may vary from tens of seconds to several hours [5, 6, 8-10].

It follows from experiment [5, 6, 8, 10] (see also the review in [9]) that relaxational phenomena are most noted in the filtration of heavy petroleum oils with an increased content of asphaltenes, resins, paraffins, etc. This allows the causes of relaxational effects to be related to silt deposition and suffosion in pores of impurity particles suspended in filtrate, as well as adsorption of polymer macromolecules at pore walls.

In [2], on the basis of microscopic represntations of the processes in pores, the influence of adsorptional effects on unsteady filtration of polymer solutions is analyzed. In [9], a relaxational phenomena due to the clogging of pore channels by viscoelastic impurity particles were considered in detail from a phenomenological perspective. However, the relation between the macroscopic characteristics of filtration and the microfeatures of the process was not analyzed in [9].

The aim of the present work is to analyze the influence of deposition and suffosion of impurity particles on the feaures of nonsteady filtration of a suspension. For maximum concentration on phenomena due to the given mechanism, it is assumed that no other factors responsible for relaxational effects are present.

To simplify the calculation further, consideration is restructed to processes in the volume of the porous medium, assuming that it is infinite and ignoring the accumulation of impurity at the boundary of the medium and the inhomogeneity of its distribution in the volume of the material. Note that the influence of such edge effects was considered in [11, 12].

With the given assumptions, filtration may be described using the orindary Darcy's law

$$V = \frac{K(\varepsilon)}{\eta} \nabla p. \tag{1}$$

The dependence of the permeability of the material on its porosity, as a rule, is fairly complex in form. Some descriptions of this dependence may be found in [13-15], for example,

below, for maximum simplicity of the mathematical calculations, it is assumed that the porosity is small, and model considerations lead to the result

$$K(\varepsilon) = k\eta\varepsilon. \qquad (2)$$

Note that Eq. (2) does not satisfy the well-known Karman–Kozeni formula according to which $K(\varepsilon) \sim \varepsilon^3$ when $\varepsilon \ll 1$. The approximation in Eq. (2) is made here with the aim of maximally simple demonstration of the model here proposed and derivation of sufficiently general qualitative results. As is clear from what follows, the choice of a more correct dependence of K on ε than that in Eq. (2) only complicates the calculations, without any qualitative change in the results.

The processes of pore closure by impurity and opening of the pores is determined by the geometry of the pores and the impurity chunks, and also by the features of their interaction with the pore walls. In the predominant majority of cases, detailed information on these characteristics is unavailable, and therefore it is natural to describe processes of deposition and suffosion by a probabilistic method. Suppose that each pore is a trap for impurity particles. Then the porous medium may be regarded as a continuum of traps, each of which may be free or occupied. If the trap is occupied, there is no filtration through the corresponding pore. Note that pore closure may occur on account of the formation of layers of adsorbed impurity microparticles at the walls [16] (in this case, the particle sizes are much less than the sizes of the pore channels) and also on account of the "plugging" of a channel by a single sufficiently large particle. The first case may be investigated on the basis of the model of [2], proposed for the analysis of the influence of adsorption of polymer molecules on the filtration of the solution which contains them. The second mechanism of deposition is considered below.

Introducing the probability W_0 of capture of at least one impurity particle by the trap and the probability W_1 of trap emptying in unit time, W_0 and W_1 depend on the geometry of the pores and particles, the particle concentration, the external-pressure gradient ∇p, the features of particle interaction with the pore walls, etc. These dependences may be established experimentally or theoretically on the basis of a model of the microstructure of the porous medium and the filtering liquid. Determining the form of W_0 and W_1 falls outside the scope of the present work. Therefore, with the aim of obtaining the fundamental results and subjecting them to physical analysis, it is assumed in the first approximation that

$$W_0 = \omega_0, \quad W_1 = \omega_1 \nabla p, \qquad (3)$$

where ω_0 and ω_1 are empirical constants. The first relation in Eq. (3) means that capture of an impurity by the pores also occurs in the absence of filtrational fluxes, and the second reflects the fact that the pores may only be unblocked on account of breakaway of the impurity chunk under the head of filtrate. The model in Eq. (3) is evidently fairly rough for quantitative calculatons. However, it allows a series of important qualitative laws to be established; they may be refined for real processes by more correct choice of Eqs. (2) and (3).

Suppose further that N_0 is the number of traps, equal to the number of pores, in unit volume of the medium; N is the number of free traps; $N_0 - N$ is the number of occupied traps; and β is the mean pore volume. In this notation

$$\frac{dN}{dt} = (N_0 - N)\omega_1 \nabla p - N\omega_0, \quad V = k\varepsilon\nabla p, \quad \beta N = \varepsilon. \qquad (4)$$

The first relation in Eq. (4) means that the change in free-trap concentration in unit time occurs on account of the emptying of occupied traps and the capture of impurity by free taps. The second relation is Darcy's law in Eq. (1) after taking account of Eq. (2), and the third is the definition of the porosity of the medium in terms of the pore concentration and their mean volume. Substitution of the third relation in Eq. (4) into the first gives

$$\frac{d\varepsilon}{dt} = (\varepsilon_0 - \varepsilon)\omega_1 \nabla p - \varepsilon\omega_0, \quad V = k\varepsilon\nabla p, \quad \varepsilon_0 = \beta N_0, \qquad (5)$$

where ε_0 is the porosity available for filtration, when all the pore–traps are free. It is evident from Eq. (5) that each steady value of the pressure gradient ∇p corresponds to a particular steady value of the porosity, and

$$\varepsilon_c = \varepsilon_0 \frac{\omega_1 \nabla p}{\omega_1 \nabla p + \omega_0}. \qquad (6)$$

There is no fundamental difficulty in using Eq. (5) to analyze the dependence $V(t)$ with arbitrarily varying ∇p. For the sake of simplicity, the situation considered below is one often encountered in practice and in experiments, in which there is a jump in the external-pressure gradient ∇p (or the flux V) from some initial value ∇p° (or V°) to the final value $\nabla p'$ (or V') at some time t° ($t^\circ = 0$ is assumed below). The corresponding values of ε_c calculated from Eq. (6) are denoted by ε_c° and ε_c'.

Jump in Pressure Gradient

In this case, it follows from the first relation in Eq. (5) that

$$\varepsilon = \varepsilon_c' + (\varepsilon_c^\circ - \varepsilon_c') \exp(-t/\tau'), \quad \tau' = (\omega_1 \nabla p' + \omega_0)^{-1}. \tag{7}$$

Using Eq. (7) in the second relation of Eq. (5), it is found that

$$V = k\varepsilon_c' \left[1 - \left(1 - \frac{\varepsilon_c^\circ}{\varepsilon_c'} \right) \exp(-t/\tau') \right] \nabla p. \tag{8}$$

This result is now compared with that following from the phenomenological equations introduced in [4-7]. Considering the case when the flux is described by a single relaxation time τ_V, the equations of [4-7] are written in the form

$$\left(1 + \tau_V \frac{\partial}{\partial t} \right) V = k\varepsilon \nabla p. \tag{9}$$

Assuming that τ_V, ε = const as in [4-7], it follows from Eq. (9) with a discontinuous change in ∇p from ∇p° to $\nabla p'$ that

$$V = k\varepsilon [1 - \exp(-t/\tau_V)] \nabla p. \tag{10}$$

It is evident that Eqs. (8) and (10) formally coincide except for some preexponential factor. Setting ∇p°, $\varepsilon_c^\circ = 0$ in Eq. (8) (i.e., assuming that the liquid is at rest before the onset of discontinuity), Eqs. (8) and (10) completely coincide formally. Note, however, that if in Eq. (10), as in [4-7], τ_V is assumed to be a constant parameter intrinsic to the system, the relaxation time τ' in Eq. (8), as is evident from Eq. (7), depends on the final pressure gradient $\nabla p'$: τ' decreases with increase in $\nabla p'$. This means, in particular, that, if the pressure gradient increases initially from ∇p° to $\nabla p'$ and then returns to ∇p°, the time to establish a steady state in the second case is greater than in the first. In experiments, this may be interpreted as a hysteresis-type dependence of V on ∇p. Note that such hysteresis was recorded experimentally in [8, 10].

The steady (as $t \to \infty$) dependence of V on ∇p is now investigated. Simple analysis of Eqs. (6) and (8) shows that, in this case, the dependence $V = k\varepsilon_c \nabla p$ takes the form shwon schematically in Fig. 1. If the experimental conditions are such that their results lie on the curve in Fig. 1 close to the asymptote, which is the case when $\omega_1 \nabla p \gg \omega_0$, extrapolation to small ∇p gives an intercept on the abscissa of length equal to ω_0/ω_1. This may be interpreted as the presence of a limiting pressure gradient.

Discontinuity in Flow Rate

Now suppose that the liquid flow rate changes discontinuously from V° to V', and the pressure gradient relaxes to a variable state.

Substituting the second relation in Eq. (5) into the first, integration leads to an equation for determining ε

$$(\varepsilon^2 + \varepsilon\gamma - \varepsilon_0\gamma) \left(\frac{\varepsilon + \Omega_1}{\varepsilon + \Omega_2} \right)^\alpha = (\varepsilon_c^2 + \varepsilon_c^\circ \gamma - \varepsilon_0\gamma) \left(\frac{\varepsilon_c^\circ + \Omega_1}{\varepsilon_c^\circ + \Omega_2} \right)^\alpha \exp(-\omega_0 t),$$

$$\gamma = \frac{\omega_1}{k\omega_0} V', \quad \alpha = \frac{\gamma}{\sqrt{\gamma^2 + 4\varepsilon_0\gamma}},$$

$$\Omega_1 = \gamma - \sqrt{\gamma^2 + 4\varepsilon_0\gamma}, \quad \Omega_2 = \gamma + \sqrt{\gamma^2 + 4\varepsilon_0\gamma}, \tag{11}$$
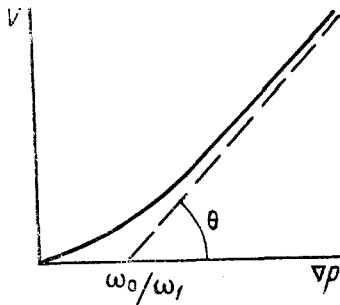
Fig. 1   Fig. 2

Fig. 1. Steady dependence of the flux V on the applied-pressure gradient ∇p according to Eqs. (4) and (5) with $\varepsilon = \varepsilon_c$ calculated from Eq. (6) (continuous curve); the dashed curve is the asymptote; $\tan \theta = k\varepsilon_0$.
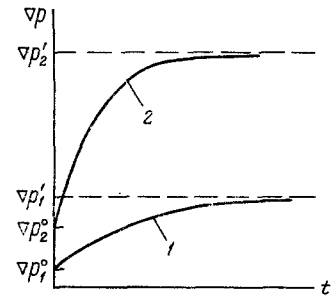
Fig. 2. Relaxation of pressure gradient ∇p to the equilibrium values $\nabla p'$ and $\nabla p'_2$ with discontinuous change in the flux V from V° to V' and V'_2 (curves 1 and 2, respectively) according to Eqs. (4) and (11): $\nabla p_i^\circ = k\varepsilon_c^\circ V^\circ$; $\nabla p_i' = k\varepsilon_c' V_i'$ ($i = 1, 2$), where ε_c°, ε_c' are calculated from Eqs. (6) with $\nabla p = k\varepsilon_c^\circ V^\circ$ and $\nabla p = k\varepsilon_c' V_i$; $V_1 < V_2$ is assumed in the calculations.

where ε_c° is the steady value of the porosity determined from Eq. (6), where ∇p is replaced by the steady pressure gradient $\nabla p^\circ = V^\circ/k\varepsilon_c^\circ$ corresponding to the initial flow-rate value V°.

The algebraic Eq. (11) may only be solved numerically. The results of numerical calculation of the dependence of ∇p on t are shown schematically in Fig. 2. Note that, as is evident from Eq. (11), the relaxation time in this case if $\tau'' = \omega_0^{-1}$. Here $\tau'' > \tau'$, where τ' is determined from Eq. (7), and therefore with discontinuous change in the flow rate steady conditions of filtration are established more slowly than with jump in the pressure. This conclusion is qualitatively confirmed by the experimetnal results of [8, 10].

Note that, as follows from Eqs. (2) and (6), the steady permeability $k\varepsilon_c$ corresponding to steady flow increases with increase in ∇p and hence V. In [2], in analyzing the filtration of polymer solution, the opposite conclusion was reached. This difference is explained in that the growth rate of the polymer adsorbate covering the pore channels and the intensity of "clogging" of the pores by impurity particles depend differently on the flow rate of filtrate. In addition, no account was taken in [2] of the possibility of breakdown of the polymer layer at the pore walls and hence of opening of the pore channels on account of hydrodynamic stress in the filtering liquid.

The quantity ω_0 determined in Eq. (3) is the probability of capture of at least one particle by the trap. In solving a series of problems, it is more expedient to take the probability of capture of a single isolated particle ω_0^* as the basis, isolating the explicit dependence on the impurity-particle concentration in ω_0.

Suppose that there are n particles in unit volume of liquid. Then, in the pore volume, the corresponding number is βn. Assuming that all the particles are the same, and using the well-known results of probaiblity theory, the following relation is obtained

$$\omega_0 = \beta n \omega_0^*. \tag{12}$$

If Eq. (12) is now substituted into Eqs. (7) and (11), the deplendence of the relaxation time on the impurity-particle concentration in the filtrate is estimated.

Thus, the proposed model of a continuum of traps leads sufficiently simply to an important qualitative conclusion regarding the features of nonsteady filtration due to silt deposition and suffosion of impurity: these processes lead to relaxational phenomena; the characteristic times are, generally speaking, operational parameters and depend on the impurity orocess in the filtrate. Steady filtration does not conform to Darcy's law and is nonlinear in character. Within the framework of the given model, it is simple to obtain quantitative results. This entails replacing Eqs. (2) and (3) by more accurate relations — for example, the Karman—Kozeni formula in place of Eq. (2). The difficulties arising here are exclusively of computational type.

Regrettably, the known experimental data on relaxational filtration of petroleum oils do not permit unique determination of the quantities W_0, W_1, ω_0, ω_1, and ω_0^* introduced here nor the elucidation of whether these relaxational phenomena are processes of deposition and suffosion. Goal-directed experiments are required to resolve these questions, and their results must be compared with estimates obtained from theoretical calculations. However, if it is known a priori that disruption of Darcy's law may be explaiend solely by blocking of the pores by impurity (as, for example, in the flow of suspensions through finely porous filters), the filtration process may be completely described using the method here proposed.

Note, in conclusion, that, as follows from Eq. (5), filtration ceases instantaneously ($V = 0$) when ∇p vanishes, within the framework of the proposed model. This contradicts the well-known data on the filtration of petroleum oils and polymer solutions, and indicates that deposition and suffosion do not determine all the relaxational effects. Sufficiently complete description of relaxational filtation entails theoretical and experimental investigation of the features of elastoviscous behavior of liquids with impurities in small pores and narrow slots, as well as in the interaction of real liquids wiht a solid frame, which will form the subject of separate investigations.

NOTATION

$K(\varepsilon)$, permeability; k, coefficient introduced in Eq. (2); p, pressure; V, flux; t, time; ε, porosity; ε_0 and ε_c, as defined in Eqs. (5) and (6), respectively; ε° and ε', steady porosity values before and after pressure (or flow) discontinuity; η, viscosity of filtrate; τ', as defined in Eq. (7); ω_0 and ω_1, as defined in Eq. (3); ω_0^*, probability of the capture of one particle in unit time.

LITERATURE CITED

1. G. T. Barenblatt, Yu. G. Mamedov, A. Kh. Mirzadzhanzade, and I. A. Shvetsov, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5, 76-83 (1973).
2. V. M. Entov and A. M. Polishchuk, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3, 68-76 (1975).
3. K. V. Mukuk, Elements of the Hydraulics of Relaxing Anomalous Systems [in Russian], Tashkent (1980).
4. I. M. Alizade, I. M. Ametov, and R. B. Mamadzade, Izv. Vyssh. Uchebn. Zaved., Neft' Gas, No. 9, 52-54 (1975).
5. Zh. A. Akilov, Izv. Akad. Nauk UzSSR, Ser. Tekh. Nauk, No. 5, 50-54 (1982).
6. Zh. A. Akilov, Dokl. Akad. Nauk UzSSR, No. 4, 20-23 (1983).
7. Yu. M. Molokovich, Relaxational Filtration [in Russian], Kazan (1980).
8. B. Khuzhaerov, in: Abstracts of the Proceedings of the Republican Conference of Young Scientists on the Mechanics of Anomalous Liquids [in Russian], Tashkent (1978), Part 1, p. 80.
9. V. M. Entov, in: Machines and Technology for Processing Rubber, Polymers, and Resinous Mixtures [in Russian], Yaroslavl' (1977), Part 1, pp. 14-27.
10. Zh. A. Akilov and B. Khuzhaerov, in: Abstracts of the Proceedings of the Second All-Union Conference on the Mechanics of Anomalous Systems [in Russian], Baku (1977), pp. 77-78.
11. A. A. Belyi, R. M. Éigel's, and A. F. Él'kind, Inzh.-Fiz. Zh., 52, No. 1, 25-32 (1987).
12. V. A. Zhuzhikov, Filtration [in Russian], Moscow (1980).
13. M. É. Aérov and O. M. Todes, Hydraulic and Thermal Operating Principles of Equipment with a Steady and Fluidized Bed [in Russian], Leningrad (1968).
14. G. Happel and G. Brenner, Hydrodynamics at Small Reynolds Numbers [Russian translation], Moscow (1976).
15. A. E. Sheidegger, Physics of Liquid Flow through Porous Media [Russian translation], Moscow (1960).
16. I. L. Markhasin, Physicochemical Mechanics of Petroleum Beds [in Russian], Moscow (1977).